Expanded Treatment of the Time-Shift Property of FKF transform with Supply-Chain Applications

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Abstract

This paper develops a distributional theory for the two-sided Fourier–Kontorovich–Lebedev (FKF) transform and demonstrates its relevance to the analysis of modern supply-chain systems. The FKF transform combines the classical Fourier transform in a temporal variable with the Kontorovich–Lebedev transform (built upon the Macdonald function of purely imaginary order) in a positive scale or intensity variable. After recalling the underlying testing-function space and the extension of the transform to generalized functions, we establish the principal operational properties, with particular emphasis on the time-shift property. A time delay appears solely as a multiplicative phase factor in the Fourier spectral variable, leaving the magnitude spectrum invariant. This phase-only signature is shown to be especially useful for the detection and characterization of lead-time delays, transportation lags, and disruption propagation in supply chains. We then outline a conceptual FKF-based workflow that maps multi-dimensional supply-chain signals (demand, inventory, distance, intensity) into a joint frequency–scale domain, enabling spectral filtering, anomaly detection, and resilience assessment. Assumptions, computational limitations, and directions for future research are discussed. The framework bridges classical distributional transform theory with contemporary digital-twin and resilience-oriented supply-chain analytics.

Keywords: FKF transform; Macdonald function; distributional transforms; time-shift property; phase spectrum; supply-chain resilience; lead-time analysis; spectral methods; digital twins

1. Introduction

Contemporary supply-chain systems are characterized by the simultaneous presence of oscillatory temporal dynamics (seasonal demand, production cycles, disruption waves) and positive-valued scale variables (inventory levels, transportation distances, order intensities, price indices). Classical Fourier analysis captures the former but is ill-suited to the latter; purely scale adapted transforms such as the Mellin or Kontorovich–Lebedev transforms capture the latter but ignore temporal structure. The FKF transform offers a natural joint spectral representation that respects both geometries.

The Kontorovich–Lebedev (KL) component employs the Macdonald function K_{iq} of purely imaginary order, which is the eigenfunction of a second-order differential operator adapted to the half-line. When combined with the ordinary Fourier kernel in the time variable, the resulting product kernel generates a transform whose operational calculus is particularly convenient for linear differential and difference equations that mix temporal delays with positive-variable operators.

In the distributional setting pioneered by Zemanian and further developed for specialized kernels by Gudadhe, Padmaja and others, the FKF transform can be defined rigorously on a dual of a carefully constructed testing-function space. This permits the treatment of impulsive signals, discontinuous demand shocks, and other generalized functions that routinely appear in supply-chain models.

The principal contribution of the present work is twofold. First, we collect and expand the fundamental properties of the FKF transform, giving special attention to the time-shift (delay) property and its spectral interpretation. Second, we articulate why these properties render the transform relevant to supply-chain analytics, and we sketch a conceptual workflow that could be realized inside digital-twin or resilience-monitoring platforms.

2. Literature Review

The distributional theory of integral transforms with special-function kernels has a long history. Zemanian's systematic treatment of Fourier and Laplace transforms of generalized functions provided the foundational template. Subsequent work extended the approach to the Hankel, Meijer, and Kontorovich–Lebedev transforms. Gudadhe and collaborators developed structure formulae, analyticity results, and testing-function spaces for the Laplace–Meijer and LK transforms [21–24]. Padmaja examined topological duals, exponential growth conditions, and unifying transforms that connect singular Cauchy problems with applications in quantum logistics and energy systems [25–29].

On the applied side, supply-chain resilience has become a central research theme. Systematic reviews and conceptual frameworks address digital twins, artificial intelligence for risk prediction, blockchain-enabled transparency, and quantum-computing approaches to logistics optimization [2, 3, 6–8, 11, 12, 14, 15, 18, 20, 30]. Lead-time variability, disruption propagation, and multi-echelon inventory dynamics are repeatedly identified as critical sources of vulnerability. Spectral and transform-domain methods, however, remain relatively under-utilized in this literature.

The present paper sits at the intersection of these two streams. It supplies a mathematically rigorous FKF framework that is simultaneously sensitive to temporal frequencies and to positive scale variables, and it indicates how the resulting spectral signatures can inform resilience analytics.

3. Distributional Setting

Let $f(t, x)$ be a suitably restricted function or generalized function with $t \in \mathbb{R}$ and $x > 0$. The two-sided FKF transform is defined by

$$F(s, q) = (FKF\ f)(s, q) = \int_{-\infty}^{\infty} \int_{0}^{\infty} f(t, x)\, e{-}^{ist}\, K_{iq}(x)\, dx\, dt$$

where s is the Fourier spectral variable, $q > 0$ is the KL spectral variable, and $K_{iq}(x)$ is the Macdonald function of imaginary order iq. The underlying domain is

$$\Omega = \{(t, x) : -\infty < t < \infty, \quad 0 < x < \infty\}$$

For distributions the transform is understood by pairing with the FKF kernel and is defined on an open region Ω_f in the (s, q)-plane.

The FKF kernel is the product

$$\Psi_{s,q}(t, x) = e{-}^{ist}\, K_{iq}(x)$$

The Fourier factor represents temporal oscillations, frequencies, periodicity, and lead-time cycles, whereas the Kontorovich–Lebedev (KL) **factor** is naturally adapted to positive scale, distance, magnitude, or intensity variables. The KL component is constructed from the Macdonald function, or modified Bessel function of the second kind,. With $\nu = iq$ this yields the spectral relation

$$(x^2 \partial_x^2 + x\partial_x - x^2) K_{iq}(x) = -q^2 K_{iq}(x),$$

which underpins the differential properties of the transform in the positive variable. This eigenvalue relation is fundamental to the KL component of the FKF transform. It shows that the differential operator

acts diagonally on the Macdonald kernel, with eigenvalue (-q^{2}). Consequently, differential operations in the positive (x)-variable can be translated into algebraic operations in the KL spectral variable (q). This property is particularly useful when modelling supply-chain variables such as inventory scale, transportation distance, demand magnitude, production capacity, safety-stock level, or disruption intensity. To formulate the transform for generalized functions, one introduces an appropriate space of testing functions on the half-plane. Testing functions are infinitely differentiable on Ω and controlled by a countable family of seminorms of the schematic form

$$\gamma_{\{a,b,\alpha,k\}}(\varphi) = sup\_\{(t, x) \in \Omega\}\, |x^k\, D_t^{\{k_1\}} D_x^{\{k_2\}}\, \varphi(t, x)|\, \eta_{\{a,b,\alpha,k\}}(x)$$

The resulting space is a sequentially complete, countably multinormed, locally convex Hausdorff space that contains the compactly supported smooth functions. Its dual therefore contains the ordinary distributions (in the Zemanian sense), allowing the FKF transform to be extended to a wide class of generalized functions. Thus, the distributional FKF framework extends the transform beyond classical integrable functions and establishes a rigorous foundation for analyzing multidimensional supply-chain signals containing oscillations, delays, scale effects, discontinuities, and other generalized behaviors.

4. Key Characteristics of the FKF Transform

4.1 Shift in the t-Domain

The time-shift (or translation) property is one of the most operationally useful features of the FKF transform. It isolates pure temporal delay as a unimodular phase factor while leaving every other spectral attribute magnitude, KL scale structure, and the absolute frequency content completely unchanged. This separation is the mathematical foundation for the supply-chain applications developed in Section 5.

4.1.1 Statement of the property

Let $t_0 \in \mathbb{R}$ be a fixed delay and let $f(t,x)$ belong to the testing-function space (or to its dual). Define the time-shifted function by

$$f_{t_0}(t,x) := f(t - t_0, x)$$

Then the FKF transform satisfies the exact identity

$$(FKF\ f_{t_0})(s,q) = e^{i\,s\,t_0} \cdot (FKF\ f)(s,q)$$

for every spectral point (s,q) at which the transform is defined. Equivalently, in operator notation,

$$FKF \circ \tau_{t_0} = M_\varphi \circ FKF,\ \ where\ \ \varphi(s) = e^{i\,s\,t_0}\ and\ \tau_{t_0}\ is\ the\ translation\ operator.$$

4.1.2 Proof for ordinary functions

Assume first that f is integrable with respect to the product measure $dt\,dx$ against the growth of the Macdonald kernel (so that the defining integral converges absolutely). Substitute the change of variable $u = t - t_0$ into the double integral:

$$(FKF\ f_{t_0})(s,q) = \iint f(u,x)\, e^{i\,s\,(u+t_0)}\, K_{iq}(x)\, du\, dx$$

The exponential factors as

$$e^{i\,s\,(u+t_0)} = e^{i\,s\,t_0} \cdot e^{i\,su}$$

Because the phase factor $e{-}^{i\,s\,t_0}$ does not depend on the integration variables, it factors out of the integral, leaving precisely $e{-}^{i\,s\,t_0} \cdot (FKF\ f)(s,q)$. The Macdonald kernel is independent of the temporal variable and is therefore unaffected by the substitution. The Jacobian of the translation is identically one and the real line is invariant under translation, so the domain of integration remains unchanged.

4.1.3 Extension to distributions

When f is a distribution of finite order, the translation $\tau\_\{t_0\}\, f$ is defined by duality:

$$\langle \tau_{t_0} f\,, \varphi \rangle = \langle f\,, \tau_{-t_0}\, \varphi \rangle$$

for every testing function φ. Because the testing function space is invariant under temporal translations (the seminorms control all derivatives uniformly), the translated testing function remains admissible. The FKF transform of a distribution is defined by continuous extension (or by pairing against the kernel viewed as a testing function of the dual variables). Consequently, the algebraic identity continues to hold in the distributional sense. In particular it applies to Dirac combs, step discontinuities, and other impulsive signals that model sudden demand shocks or instantaneous replenishment events.

4.1.4 Magnitude invariance and phase extraction

Taking the modulus of both sides of the shift identity immediately yields

$$|\,(FKF\ f_{t_0})(s,q)\,| = |\,(FKF\ f)(s,q)\,|$$

for all (s,q). Thus a pure temporal delay is invisible to any magnitude-based spectral statistic. The entire information about the delay is carried by the argument (phase):

$$arg(\,(FKF\ f_{t_0})(s,q)\,) = arg(\,(FKF\ f)(s,q)\,)^{s\,t_0}\,(mod\ 2\pi)$$

If a baseline (un-delayed) spectrum is known either from historical data under normal operating conditions or from a calibrated digital-twin simulation the delay can be recovered by a linear regression of the unwrapped phase difference against the frequency variable s. The slope of that regression is precisely $-t_0$. Because the relation is linear in s, standard phase-unwrapping algorithms followed by ordinary least-squares estimation produce a statistically efficient estimator of the lag.

4.1.5 Multiple discrete delays and linear combinations

Suppose a signal is a finite superposition of delayed replicas,

$$g(t,x) = \textstyle\sum_{k=1} N^{\alpha_k}\, f(t - t_k, x)$$

with complex amplitudes α_k. Linearity together with the elementary shift property produces

$$(FKF\ g)(s,q) = (\,\textstyle\sum_{k=1} N^{\alpha_k}\, e^{i\,s\,t_k}\,) \cdot (FKF\ f)(s,q)$$

The term in parentheses is a trigonometric polynomial in the frequency variable s. Its zeros and peaks encode the set of delays $\{t_k\}$. In the supply-chain setting this representation corresponds to a finite collection of parallel lead-time paths (e.g., alternative suppliers or multi-modal transportation options) whose relative weights and lags can be identified from the oscillatory structure of the phase spectrum.

4.1.6 Continuous delay distributions (random lead times)

When the delay itself is a random variable with probability density $\rho(\tau)$ (or, more generally, a positive measure), the observed signal is the convolution

$$g(t,x) = \int_{\mathbb{R}} \rho(\tau)\, f(t-\tau,x)\, d\tau$$

The FKF transform converts the convolution into multiplication by the characteristic function (Fourier–Stieltjes transform) of the delay law:

$$(FKF\ g)(s,q) = \hat{\rho}(s) \cdot (FKF\ f)(s,q), \quad where \quad \hat{\rho}(s) = \int e^{is\tau}\, \rho(\tau)\, d\tau$$

Thus the magnitude spectrum is modulated by $|\hat{\rho}(s)|$ while the phase is shifted by $arg\ \hat{\rho}(s)$. For a deterministic delay the characteristic function is a pure exponential and the magnitude remains unperturbed; any randomness in lead time produces a characteristic attenuation of high frequency components. This furnishes a spectral diagnostic that distinguishes pure (deterministic) lag from stochastic lead time variability an important distinction for inventory-policy design.

4.1.7 Relation to group delay and differentiation

Differentiating the phase of the shifted spectrum with respect to frequency recovers the group delay:

$$-\frac{d}{ds}\arg\Big(\big(FKF\ f_{t_0}\big)(s,q)\Big) = t_0 + \left(-\frac{d}{ds}\arg(FKF\ f)\,(s,q)\right)$$

The second term on the right is the intrinsic group delay of the original process; the constant offset is exactly the external temporal shift. Consequently, once the intrinsic phase of a baseline process is known, any additional constant group delay is immediately attributable to an external lag. The same differentiation property also recovers the classical differentiation theorem

$$FKF(\partial_t f) = i\, s \cdot (FKF\ f)$$

which is the infinitesimal generator of the translation group and confirms consistency of the operational calculus.

5. Applications of the Shift-in-t Property to Supply-Chain Systems

The phase-only signature of a pure temporal delay makes the FKF transform a natural instrument for several core supply-chain diagnostics. We elaborate the principal use-cases below.

5.1 Lead-time estimation via phase regression

In a multi-echelon network, the effective lead time between two nodes is rarely known with precision; it fluctuates with congestion, customs clearance, carrier reliability and weather. Let $f(t,x)$ be a baseline demand or shipment intensity observed under nominal conditions, and let $g(t,x)$ be the same process observed after an unknown lag t_0. Their FKF spectra satisfy

$$(FKF\ g)(s,q)\ /\ (FKF\ f)(s,q) = e^{i\, s\, t_0}$$

wherever the baseline spectrum does not vanish. Taking the argument and unwrapping yields a linear function of s whose slope is $-t_0$. A robust linear fit (e.g., Theil–Sen or iteratively re-weighted least squares) therefore returns an estimate of the lag together with a confidence interval. Because the KL variable q does not appear in the phase factor, the regression can be performed independently on every fixed -q slice and the resulting estimates pooled, increasing statistical power and providing a consistency check across scales.

Figure 1. Lead-time estimation via phase regression

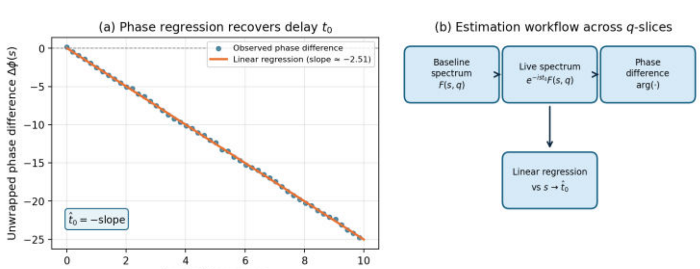

Figure 1. Lead-time estimation via phase regression: unwrapped phase difference is linear in frequency s with slope $-t_0$ (a); the estimation workflow operates independently on each KL scale slice (b).

Practical advantages over pure time domain cross correlation include:
Invariance of the magnitude spectrum protects the estimator against amplitude fluctuations caused by demand spikes or partial shipments.
The joint (s, q) representation permits the analyst to discard noisy high-q bands where the Macdonald kernel decays rapidly, improving the signal-to-noise ratio of the phase estimate.
Digital-twin residuals can be monitored continuously: any systematic drift of the estimated t_0 away from the twin's nominal lead time triggers an early-warning alert.

5.2 Distinguishing pure delay from magnitude-altering distortions

Not every disruption is a pure lag. Quality problems, partial fulfilment, or capacity loss alter the amplitude of the process as well as its timing. Because a pure shift leaves the magnitude spectrum invariant, any statistically significant change in $|FKF\ g|$ relative to $|FKF\ f|$ signals that the disturbance is not a pure temporal translation.

Figure 2. Distinguishing pure delay from magnitude-altering distortions

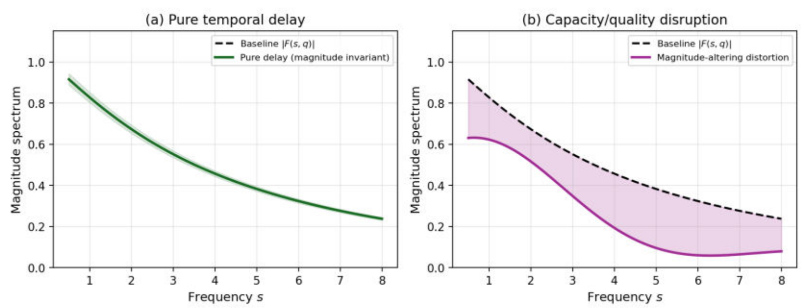

Figure 2. Distinguishing pure delay from magnitude-altering distortions: a pure temporal shift leaves the magnitude spectrum invariant (a), whereas capacity or quality disruptions produce measurable magnitude changes (b).

Conversely, a large phase shift accompanied by an essentially unchanged magnitude spectrum is diagnostic of a pure lead-time excursion. This binary classification is valuable for root-cause analysis: pure delays call for logistics interventions (expediting, alternative routing), whereas magnitude changes call for capacity or quality interventions.

5.3 multi-echelon disruption propagation as cascaded phase factors

Consider a linear cascade of n echelons in which a disruption originating at echelon 0 reaches echelon k after a cumulative delay $T_k = t_1 + \cdots + t_k$. The spectrum observed at echelon k is therefore $\Delta\varphi_k$

$$(FKF\ g_k)(s,q) = e^{i\,s\,T_k} \cdot (FKF\ g_0)(s,q)$$

The successive phase increments $\Delta\varphi_k(s) = -s\,t_k$ can be recovered by comparing adjacent echelons. Mapping the estimated delays onto the physical network yields a quantitative picture of disruption propagation speed and of the relative contribution of each inter-echelon link. Because the same phase information is available at every scale q, one can further examine whether the delay is uniform across intensity or distance bands, or whether high-intensity shipments experience systematically different lags.

Figure 3. Multi-echelon disruption propagation as cascaded phase factors

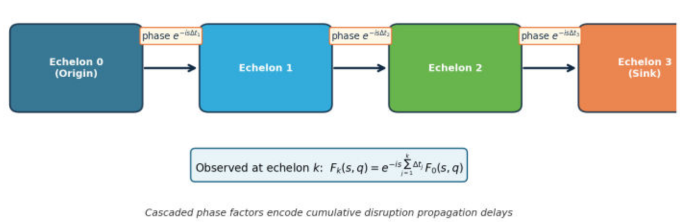

Figure 3. Multi-echelon disruption propagation: successive inter-echelon delays appear as cascaded phase factors e^{−is Δt_j}; the cumulative phase recovers the total propagation lag.

5.4 Stochastic lead-time variability and inventory policy

When lead times are random, Section 4.1.6 shows that the magnitude spectrum is attenuated by the modulus of the characteristic function of the delay distribution. High-frequency components of demand are therefore preferentially damped. Inventory policies that rely on accurate high-frequency forecasts (for example, just-in-time replenishment) become less effective precisely when lead-time variance rises. Monitoring the ratio

$$|\,(FKF\ g)(s,q)\,|\,/\,|\,(FKF\ f)(s,q)\,|\ \approx\ |\hat{\rho}(s)|$$

across a rolling window supplies a real-time indicator of lead-time dispersion. An increase in high-frequency attenuation can trigger a switch from lean to buffered inventory policies, or an adjustment of safety-stock formulae that explicitly incorporate the estimated characteristic function.

Figure 4. Stochastic lead-time variability and inventory-policy implications

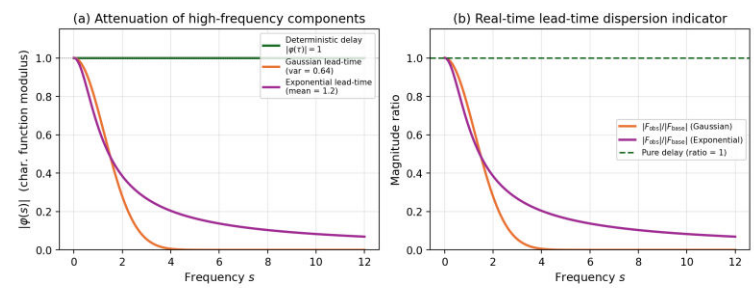

Figure 4. Stochastic lead-time variability: characteristic-function modulus attenuates high-frequency components (a); the magnitude ratio serves as a real-time dispersion indicator for inventory-policy switching (b).

5.5 Parallel paths and multi-sourcing

When several suppliers or transportation modes operate in parallel, the observed spectrum is a linear combination of phase factors (Section 4.1.5). The resulting trigonometric polynomial can be inverted by Prony-type or annihilating-filter methods to recover the individual delays and the relative volumes flowing through each path. This spectral un-mixing is especially useful for multi-sourcing strategies: an abrupt change in the relative weights of the phase terms signals a shift in supplier mix or a re-routing decision, even before the change becomes visible in aggregate volume statistics.

Figure 5. Parallel paths and multi-sourcing (spectral un-mixing)

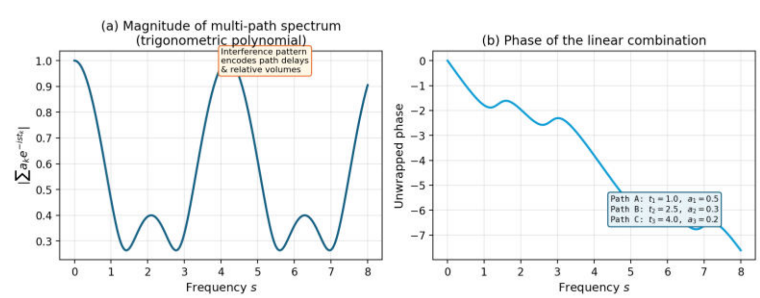

Figure 5. Parallel paths and multi-sourcing: magnitude (a) and phase (b) of a linear combination of delayed path contributions form a trigonometric polynomial that encodes individual delays and relative volumes.

5.6 Continuous residual monitoring inside digital twins

A digital twin that maintains a live FKF spectrum of the simulated "should-be" process can compare it, in real time, with the spectrum of the observed process. The residual phase

$$\Delta\varphi(s,q) = \arg(FKF_{obs}) - \arg(FKF_{twin})$$

is expected to be zero (or a known small bias) under normal conditions. Persistent linear trends in $\Delta\varphi$ versus s indicate an emerging lag; non linear phase residuals indicate more complex distortions. Because the comparison is performed entirely in the spectral domain, it is robust to the absolute scale of the signals and can be updated with every new data window without re-running a full time-domain simulation.

Figure 6. Continuous residual monitoring inside digital twins

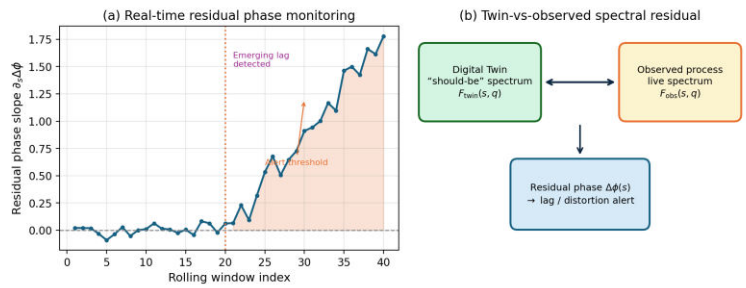

Figure 6. Continuous residual monitoring inside digital twins: residual phase slope over rolling windows (a) and conceptual twin-versus-observed spectral comparison (b).

6. Conceptual FKF-Based Supply-Chain Workflow

A practical pipeline that exploits the shift property may be organized as follows:

Signal acquisition. Collect synchronized time series of demand, inventory, shipments and a positive auxiliary variable (distance, intensity, cost).

Baseline spectrum. Compute the FKF transform of a reference window regarded as the "normal" regime (or of a digital-twin output).

Live spectrum. Recompute the FKF transform on successive rolling windows of the live data stream.

Phase analysis. Unwrap the phase difference, regress against s to estimate delays, monitor magnitude ratios to detect non-pure-delay distortions, and track multi-path trigonometric structure.

Decision support. Feed estimated lags, attenuation profiles and path weights into inventory, routing and expediting modules; raise alerts when residual phase drifts beyond prescribed thresholds.

7. Important Assumptions and Limitations

The framework rests on several idealizations:

The temporal variable ranges over the whole real line (or a sufficiently long observation window that boundary effects are negligible).

The scale variable is strictly positive; zero-inventory or zero-distance singularities must be treated by a limiting procedure or by a different kernel.

Growth and decay conditions implicit in the testing-function seminorms must be satisfied, or the signals must be interpreted as tempered distributions of appropriate order.

Numerical evaluation of the Macdonald function of imaginary order, especially for large |q| or extreme x, remains computationally non-trivial; accurate and fast libraries are still an active research topic.

Phase unwrapping can become ambiguous when the signal-to-noise ratio is low or when the baseline spectrum has deep nulls; robust unwrapping algorithms and regularization are required.

The inverse FKF transform, while theoretically existent under suitable conditions, lacks a simple closed-form expression comparable to the classical Fourier inversion formula; practical reconstruction therefore relies on numerical quadrature or series expansions.

8.1 Recommendations for research and practice

Develop fast, stable numerical algorithms for the forward and inverse FKF transforms, leveraging asymptotic expansions of the Macdonald function and modern GPU-accelerated quadrature.

Construct explicit testing-function spaces tailored to the growth rates observed in real supply-chain data (heavy-tailed demand, exponential inventory decay, etc.).

Implement robust phase-unwrapping and linear-regression pipelines that operate on rolling FKF spectra and publish open benchmark data sets with known injected delays.

Embed the FKF front-end inside existing digital-twin platforms and quantify the incremental improvement in delay-estimation accuracy and anomaly-detection lead time.

Investigate the interaction of the FKF spectrum with quantum-annealing and other combinatorial optimizers used for vehicle routing and inventory allocation.

8.2 Conclusion

The FKF transform furnishes a mathematically coherent joint spectral representation for signals that live simultaneously on the real line and on the positive half line. Its distributional extension accommodates the impulsive and discontinuous features typical of supply-chain data. The time shift property supplies a clean phase signature of pure temporal delay, while the Macdonald factor resolves positive scale structure. The expanded analysis of the shift property covering deterministic lags, multiple discrete paths, continuous delay distributions, magnitude invariance and group-delay extraction opens a concrete suite of diagnostic tools for lead-time monitoring, disruption propagation tracking, multi sourcing un mixing and digital-twin residual analysis. Realization of these tools will require further numerical, theoretical and empirical work; the conceptual and mathematical foundation, however, is already in place.

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